of the pattern-ray, and P_1PP_2 for the plane on which the shadow is cast. Draw through a_1, a_2 the lines a_1p_1, a_2p_2 parallel to R_1, R_2; these will be the projections of the line of shadow, and a_1p_1 (found as before taught in problem A) will be its traces on the plan and elevation planes respectively. Moreover, the planes $a_1\beta_1\beta_2$, $a_2\beta_2a_1$ are the projecting planes of that line through a_1a_2 parallel to R_1R_2; and hence are particular and convenient cases of the planes, Q_1QQ_2, R_1RR_2 of the eidograph (fig. 8) for our present purpose. Also, since P_1P_2 are the horizontal and vertical traces of the intersection of the plane $P_1\beta_1P_2$ through the line of shadow with the plane P_1PP_2, which is to receive the shadow, the line P_2p is the vertical projection of that intersection, and contains the vertical projection of the point sought. The same point is also in a_2a_2, the vertical projection of the shadow-line itself. Whence their intersection, p_2, is the vertical projection of the shadow of a_1a_2 on P_1PP_2.

Again, $a_1\beta_1$ is the horizontal projection of the line pp_1, and hence contains the horizontal projection of the shadow of a_1a_2 on P_1PP_2. The projections of that point, too, are in a perpendicular p_2p_1 to OX, as is shewn in (I.). Whence follows

THE SYNTHESIS.—Draw through a_1a_2 the lines $a_1\beta_1$, $a_2\alpha_2$, to meet the ground line OX; from β_1 (or α_2, here taken β_1 as may be most convenient in the individual case) draw a perpendicular β_1P_2 to meet PP_2 in P_2; draw P_2p_1, also perpendicular to OX, and join pP_2, meeting $a_2\alpha_2$ in p_2; and, finally, draw p_2p_1 perpendicular to OX, meeting $P_1\beta_1$ in p_1. Then p_1p_2 are the plan and elevation of the shadow of a_1a_2 on P_1PP_2.

Note.—If the perpendicular from a_2 had been used, the final result would have been the same; only the point p_1 would have been first found, and thence the point p_2, in an order the reverse of that specifically detailed above. The work for finding it in this order is marked by broken lines and dots, thus :—.—.—.—. The student is recommended to practise each method separately.

VII. The preceding construction is perfectly general, and applicable to all possible cases of the relative positions of the point and plane, and the direction of light. It meets, therefore, the greatest difficulty encountered *in practice* in drawing the projections of a shadow upon a plane; viz., that of shadows on roofs. When the shadows are to fall upon planes perpendicular to the horizon or to the vertical plane, the process becomes more simple. I shall give the modified constructions adapted to both cases.

(1.) *Fig.* 10.—Let the plane P_1PP_2 which receives the shadow be vertical.

Draw the horizontal and vertical projections of the line of shadow through a_1, a_2 parallel to R_1, R_2, and let the horizontal projection a_1p_1 meet the horizontal trace PP_1 of the given plane P_1PP_2 in p_1; and draw p_1pp_2 perpendicular to OX to meet the vertical projection a_2p_2 in p_2. Then p_1, p_2 are the plan and elevation of the shadow of a_1, a_2.

(2.) *Fig.* 11.—Let the plane P_1PP_2 be perpendicular to the elevation plane.

Let the vertical projection of the shadow-line meet the vertical trace PP_2 in p_2; and draw p_2pp_1 perpendicular to OX to meet the horizontal projection of the shadow-line in p_1. Then p_1, p_2 are the plan and elevation of the shadow of a_1, a_2.

The truth of these constructions may be deduced with great readiness from the general case, but simpler considerations will accomplish that purpose. Take, for instance, the first case.

Since the projecting plane of the shadow-line and the plane P_1PP_2 are perpendicular to the plan, their intersection is also perpendicular to the plan; and the shadow-line cuts P_1PP_2 in that perpendicular, being in one of the planes which meet in that line. The point p_1 is therefore the horizontal projection of the shadow of a_1a_2 on P_1PP_2. Also, the intersection of the vertical projection of the shadow-line with the perpendicular p_2pp_1 to OX, it being in both those lines. Whence p_1, p_2 are the plan and elevation of the point required.

VIII. We have hitherto considered the plane to be indefinitely extended, or capable of being so. Except, however, in case of the horizontal and vertical planes, or occasionally (for all practical purposes) a sloping terrace, this condition is not fulfilled in fact; and the shadow may or may not fall upon that portion of the plane which comes into the drawing. The process to be then adopted will require a specific notice.

When the plane is arbitrary in its relative position to the planes of projection, its *boundaries* are always given in plan and elevation; as, for instance, the roof of a building. The same, indeed, is true for planes perpendicular to the horizontal and vertical planes: but as in the preceding cases, these become more simple than the general problem, they will be advantageously considered by themselves. In these latter cases the traces of the planes which receive the shadow are either positively or virtually given: whilst in the general case some consideration is required, as well as a portion of work, to find those traces.

Plane roofs inclined to the horizon are almost always, if not always, either rectangular, trapezoïdal, or triangular. Should the case (contrary to my knowledge) be otherwise, the process here given equally applies.

Three points in space, not situated in a right line, determine the position of the plane in which they are; and two lines which are parallel, which meet, or which being produced would meet, also determine the plane which contains them. Now, when the plane is given by its boundaries in plan and elevation, we may select three points in it—two lines which meet, or two lines which are parallel—from which to find the traces of the plane itself. As an example, we shall take two lines which meet one another.

Fig. 12. Let them be a_1b_1, a_2b_2, and a_1c_1, a_2c_2. Find the traces $\alpha_1\beta_2$ and $\gamma_1\delta_2$ of these lines. Then, since these lines are by hypothesis in one plane (the condition being that they meet, and this condition implying that a_1a_2 is perpendicular to OX), the traces on each plane of these lines are in the respective traces of the plane through them. But the points α_1, γ_1 in the horizontal trace being given, that trace itself is given; and in like manner the vertical trace is given by the points β_2, δ_2. The construction is completed by drawing those lines, which, being in the plane, will meet OX in the same point P.

When two parallel lines are given, the process is the same, viz.: to find their traces, and draw the traces of the plane through them. Also, when three points are given, two lines through one of them and each of the two others (selected at pleasure or for convenience), or a line through two of them and a parallel to it through the third, will serve in the same way to determine the traces of the plane which passes through them.

It will be obvious, that when the work is carefully performed, one of the four traces, α_1, γ_1, β_2, δ_2, may be dispensed with, as the point P found from two of them supplies its place in determining the trace of the plane. As, however, the operation is often a delicate one, it may be advantageous to use all, and reserve the meeting of α_1, γ_1, β_2, δ_2, in OX as a test of accuracy.

Having found the traces of the plane ABCD, whose projections are $a_1b_1c_1d_1$ and $a_2b_2c_2d_2$, find by VI. the shadow of any given point upon it, viz. p_1p_2 or q_1q_2. In the figure here employed p_1p_2 falls within the boundary of the given limited plane ABCD, and q_1q_2 without that boundary. That is, the one point has a shadow on the given *finite* plane and the other has not.

Having disposed of the general case, let us consider the more familiar and simple one of shadows of points on upright walls.

Fig. 13. Let $a_1b_1c_1d_1$ be the plan, and aa_2, bb_2, cc_2, dd_2 the elevations of the several vertical edges of the building, and g_1g_2, h_1h_2, k_1k_2 the respective projections of points which cast their shadows, the pattern-ray being R_1, R_2.

The shadow of g_1 g_2 falls upon the plane whose plan is a_1d_1, and its p_1, p_2 are its plan and elevation respectively: the shadows of h_1h_2 fall upon that whose plan is a_1b_1, and is q_1, q_2: whilst the shadow of k_1k_2 does not fall upon the structure at all.

IX. In connection with this branch of the subject, it yet remains to construct the shadow of a line upon a plane, or upon any number of planes.

Now the shadow of a line is given when the shadows of two points in it are given on any other plane, as well as on those of projection, already treated in (V.). One portion of the shadow of the line may fall upon the given plane, and other parts upon other given planes, or even upon the planes of projection themselves.

Whilst the line which casts the shadow and the plane which is to receive it are of unlimited extension, the indefinite extension of line which passes through the projections of any two selected points in the given line will be the corresponding projection of the shadow. When the line which casts the shadow is limited both ways, the projections of the shadow on any one given plane will be the lines which join the projections of those extremities; and these may be found separately as in the article (VI.). However, instead of working for each point separately (which often involves a repetition of work already done), we may modify the process so as to meet every possible case, and at the same time save a considerable portion of trouble in the operation: this will require, like some former problems, that we lay down certain principles upon which to establish the process.

(*a*) If two straight lines in space be not parallel, there can be drawn through each line a plane parallel to the other line; these planes will be parallel to each other; and there can only be one such pair.

Fig. 14. Let AB, CD, be two straight lines not in one plane. Take the points E, F in them at pleasure; through E draw C'D' parallel to CD, and through F draw A'B' parallel to AB. Then the planes through these, viz. AC'BD' and A'CB'D are parallel. (*Euc. xi.* 15.) Whence the planes being parallel, no line in the one can meet the other plane: that is, the line AB is parallel to the plane A'CB'D, and CD is parallel to the plane AC'BD'. Whence the first and second parts of the proposition are proved.

Nor can there be any other pair of such parallel planes. For, take any other point, *e*, in AB, and draw *c'd'* parallel to CD. Then, since C'D', *c'd'* are parallel, and Ee joins them, the plane AC'BD' is coincident with and cannot be different from Ac'Bd'. Whence no other pair of parallels, &c.

(*b*) Having given two straight lines by their projections in plan and elevation; to find the traces of a plane through one of them parallel to the other.

Fig. 15.—Let the traces $a_1\beta_2$, $\gamma_1\delta_2$ of the given lines be found as already taught in (I.), and let it be required to draw through $a_1\beta_2$ a plane parallel to $\gamma_1'\delta_2'$.

Take any point a_1a_2 in $a_1\beta_2$, and draw parallels to $\gamma_1'\delta_1'$, $\gamma_2'\delta_2'$ to meet OX: these will be the projections of a line parallel to $\gamma_1'\delta_2'$ (III.). Find the traces γ_1, δ_2 of this line: then the lines $a_1\beta_2$, $\gamma_1\delta_2$ being in one plane, their traces are in the traces of that plane. The lines γ_1a_1, $\beta_2\delta_2$ being drawn to meet in P_1 are the traces of the plane required.

(*c*) The shadow of a line upon a plane is the intersection of a plane drawn through the line parallel to the pattern-ray with the plane which receives the shadow. We are therefore now prepared to solve the following problem in its most general form. This, however, must be reserved for the next of this series of papers.

Charlton, near Woolwich,
August 12, 1846.

STATE OF THE LONDON CHURCH-YARDS, &c.*

THE present closing session has not only been eventful, but full of *great* events—apparently designed, beyond many preceding, to shew that "out of evil comes good." And, though this is actually a minor point, if it has not sounded the "knell," it may have gone nigh to ring the "*passing*" bell of "burial in the heart of towns."

Much of the working of a system may be learnt from the bearing of the officials. On the disturbance of coffins, the writer was once told at St. Margaret's, Westminster—"Oh! what's the use of *keeping boards?*"—at St. Giles's-in-the-fields (behind the church), "If we did not *break up the old coffin wood* we should never get a grave here." Now this

* Vide p. 288, *ante*.—A correspondent says, with reference to Mr. Pusey's last letter,—"It is stated some of the new churches in Bethnal Green have any churchyard. St. Bartholomew's, Cambridge-road, has one—small, I admit—and about a month ago the Bishop of London consecrated a burial-ground attached to another of the new churches, I believe the one near Twig Folly. I sincerely hope that the time will ere long arrive when there will be no more burials in the present crowded churchyards. E. P. B.

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